

# The evolution of density perturbations in two quintessence models

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## Abstract

In this work we investigate the evolution of matter density perturbations for two different quintessence models. One of them is based on the Einstein theory of gravity, while the other is based on the Brans-Dicke scalar tensor theory. We show that it is possible to constrain the parameter space of the models using the determinations for the growth rate of perturbations derived from data of the 2-degree Field Galaxy Redshift Survey.

Key Words: cosmology: observations, cosmology: theory, large-scale structure of Universe.

## I. INTRODUCTION

In the past few years, it has become apparent that the energy budget of our universe is dominated by an unknown component called "dark energy". The WMAP table of "best" cosmological parameters [1], for instance, gives a  $0.73 \pm 0.04$  abundance for it, and a value of its equation of state  $\omega < -0.78$ . In order to relieve some problems of the popular  $\Lambda CDM$  model (like the fine tuning issue), a dynamical  $\Lambda$ -term has been proposed as representative of the dark energy. Its more popular version is a slowly rolling scalar field named quintessence. Many alternative cosmological models have been proposed, and indeed it is a challenge the work of ruling out all the "incorrect ones" on observational grounds. For instance many

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different potentials for these self interacting scalar fields (quintessence) have been proposed. However, it is obvious the importance of the observational exploration of the proposed cosmological models, and in this paper we give a further step in this direction. In [3] two cosmological models are proposed. In both cases two fluids fill the universe: a background fluid of ordinary matter and a self-interacting scalar-field fluid accounting for the dark energy, both in Einstein's theory and in Brans-Dicke gravity. A linear relationship between the Hubble expansion parameter and the time derivative of the scalar field is used to derive exact cosmological attractor-like solutions. And a priori assumptions about the functional form of the potential or the scale factor behavior are not necessary. All these features render two very interesting models that motivated us to proceed with the observational check of them. In the original paper, the authors made a first check with some observational facts, related with Cosmic Microwave Background (CMB), nucleosynthesis, structure formation (restriction on quintessence density during galaxy formation epoch) and type Ia supernovae (SN Ia). Now we proceed the observational checking considering another aspect of structure formation: the galaxy motions and clustering, i. e., the evolution of density perturbations in the Universe. In the past few years, observations of the large scale structure of the Universe have improved greatly. The development of fiber-fed spectrographs that can simultaneously measure spectra of hundreds of galaxies has provided large redshift surveys such as the 2-degree Field Galaxy Redshift Survey (2dFGRS) and the Sloan Digital Sky Survey (SDSS). In particular, the Anglo-Australian Telescope of the 2dFGRS has obtained the redshift of a quarter million galaxies. This collaboration has produced abundant data and technical papers [2] about galaxy motions and clustering, and we will refer to some of this, in particular their velocity/density comparisons.

The paper is organised as follows: in section II we outline the main characteristics of the models, in section III the main aspects of velocity/density comparisons are exposed and the equation for the growth of perturbations is solved, in section IV the observational check is presented and interpreted, while in section V conclusions are drawn. Finally, references are supplied.

## II. THE MODELS

In this section we supply the main equations characterizing the dynamics of the models. For a detailed description of them, we refer to [3]. As said above, one of the models is based on the Einstein gravity and the other in the Brans-Dicke one.

### A. Einstein Theory

We analyze the solution in Einstein's theory with two fluids: a background fluid of ordinary matter and a self-interacting scalar fluid that accounts for the dark energy component in the universe that Arias et. all. studied in [3]. In this case the ansatz relating the expansion parameter and the time derivative of the scalar field is:

$$H = k\dot{\phi}, \quad (1)$$

where  $k$  is a constant parameter. The exact solution for, respectively, the scale factor, the Hubble expansion parameter, the matter density and equation of state are

$$a(\tau) = \left\{ \sqrt{\frac{A}{B}} \sinh[\mu(\tau + \tau_0)] \right\}^{\frac{2k^2}{3k^2\gamma - 1}} \quad (2)$$

$$H(\tau) = \sqrt{\frac{8k^2}{6k^2 - 1}} \xi_0 a(\tau)^{-1/2k^2} \coth[\mu(\tau + \tau_0)] \quad (3)$$

$$\Omega_m(\tau) = (1 - \varepsilon) \{ \cosh[\mu(\tau + \tau_0)] \}^{-2} \quad (4)$$

$$\omega_\phi(\tau) = -1 + \frac{1}{3k^2(1 - \Omega_m(\tau))} \quad (5)$$

whit  $dt = a^{\frac{1}{2k^2}} d\tau$ . This cosmological model depends only on three free parameters  $(\gamma, k, \varepsilon)$ . The intermediate parameters  $A, B, \mu, \tau_0$  and  $\xi_0$  are determined from the normalisation used or recast into  $(\gamma, k, \varepsilon)$ .  $\gamma$  is the barotropic index of the matter fluid. In this paper we study the evolution of matter density perturbations, well into the epoch of matter domination over radiation, so we set  $\gamma = 1$ . As seen from (1),  $k$  is the parameter relating the expansion rate

and the time derivative of the scalar field , and  $\varepsilon$  is the density  $\Omega_\phi(z)$  of dark energy in the early stages of the evolution ( $z \sim \infty$ ) provided the universe is flat. From CMB, nucleosynthesis and galaxy formation observations, in [3] the authors additionally constrained this parameter space to be ( $\gamma = 1, k > 0, 0 \leq \varepsilon \leq 0.045$ ). Interesting to note that SN Ia was not useful to constrain the parameter space. Indeed, it is well known the controversy about the degeneracy of supernovae observations [4–9]. At this model we will call Model E (Einstein gravity).

## B. Brans-Dicke gravity

In this model we analyse the exact solution in gravity with non-minimal coupling between the quintessence field and the background fluid, in particular the Brans-Dicke theory with two fluids: a barotropic perfect fluid of ordinary matter and a self-interacting scalar fluid accounting for the dark energy in the universe. In this case we are faced with two relevant frames (the Jordan frame and the Einstein frame), in which Brans-Dicke theory can be formulated. There has been discussion on whether these two frames are equivalent [10]. It is not our aim to participate in this controversy and for practical reasons (simplicity of mathematical handling) we chose the Einstein frame. Now the ansatz relating the expansion parameter and the time derivative of the (rescaled) scalar field is:

$$\bar{H} = \frac{\dot{\psi}}{\lambda}, \quad (6)$$

where  $\lambda$  is a constant parameter. The bar notation means we are working in the Einstein frame of BD theory. In this frame, the exact solutions for, respectively, the scale factor, the matter density and equation of state are

$$\bar{a}(r) = \left\{ \sqrt{\frac{A}{B}} \sinh[\mu(r + r_0)] \right\}^{\frac{2}{\lambda(\delta - \lambda)}} \quad (7)$$

$$\bar{\Omega}_m(r) = n^2(1 - \varepsilon) \{ \cosh[\mu(r + r_0)] \}^{-2} \quad (8)$$

$$\bar{\omega}_\psi(r) = -1 + \frac{\lambda^2}{3(1 - \bar{\Omega}_m(r))} \quad (9)$$

where  $d\bar{t} = \bar{a}^{-\frac{\lambda^2}{2}} dr$ . This cosmological model also depends on three free parameters  $(\gamma, \lambda, \varepsilon)$ . Like in the former case, the intermediate parameters  $A, B, \mu, r_0, \delta$  and  $n$  are determined from the normalisation used or expressed through  $(\gamma, \lambda, \varepsilon)$ . Also, from CMB, nucleosynthesis and galaxy formation observations, in [3] the authors constrained this parameter space to be  $(\gamma = 1, 0 < \lambda < 0.37, 0 \leq \varepsilon \leq 0.045)$ . Again SN Ia observations were not useful to constrain the parameter space. To this case we will call Model BD (Brans-Dicke gravity).

### III. PERTURBATION GROWTH

We will study the evolution of mass density contrast ( $\delta = \delta\rho/\rho$ ) in the mass distribution, modelled as a pressureless fluid, in linear perturbation theory. This method is based on Newtonian mechanics, that is better suited to the study of the development of structure such as galaxies and clusters of galaxies. This computation requires that we be able to isolate a region small enough for the Newtonian gravitational potential energy and the relatives velocities within the region to be small (non relativistic) [11]. The equation of time evolution of mass density contrast is

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \quad (10)$$

Where the dot means derivative with respect to the comoving time. In this equation the relative contribution of dark energy to the energy budget enters into the expansion rate  $H$ . We shall consider this equation in the matter dominated era, when the radiation contribution is really negligible. The linear theory relates the peculiar velocity field  $v$  and the density contrast by

$$v(x) = H_0 \frac{f}{4\pi} \int \delta_m(y) \frac{x-y}{|x-y|^3} d^3y \quad (11)$$

where the growth index  $f$  is defined as

$$f \equiv \frac{d \ln \delta_m}{d \ln a} \quad (12)$$

where  $a$  is the scale factor. To solve the equation (10) for the evolution or perturbations we introduced several changes of variable, allowing to rewrite it in the form

$$X^2(1+X^2)\ddot{\delta_m} + X\dot{\delta_m}(X^2d+e) - c\delta_m = 0 \quad (13)$$

where  $X = \sinh[\mu(\tau + \tau_0)]$ , the dot means derivative with respect to  $X$ , and  $c, d$  and  $e$  are constants characteristic of each model. Equation (13) has two linearly independent solutions, the growing mode  $\delta_{m+}$  and the decreasing mode  $\delta_{m-}$ , which can be expressed in terms of hypergeometric functions of second type  ${}_2F_1$ . We get

$$\delta_{m+} \propto X^{1/2\{\sqrt{A}+1-e\}} {}_2F_1\left[\frac{1}{4} - \frac{e}{4} + \frac{1}{4}\sqrt{A}, -\frac{1}{4} + \frac{d}{2} - \frac{e}{4} + \frac{1}{4}\sqrt{A}, 1 + \frac{1}{2}\sqrt{A}, -X^2\right] \quad (14)$$

$$\delta_{m-} \propto X^{1/2\{-\sqrt{A}+1-e\}} {}_2F_1\left[\frac{1}{4} - \frac{e}{4} - \frac{1}{4}\sqrt{A}, -\frac{1}{4} + \frac{d}{2} - \frac{e}{4} - \frac{1}{4}\sqrt{A}, 1 - \frac{1}{2}\sqrt{A}, -X^2\right] \quad (15)$$

where  $A = (e-1)^2 + 4c$  and . For  $\tau \ll 1$  we can write

$$\delta_{m+} \propto X^{1/2\{1+\sqrt{A}-e\}}, \delta_{m-} \propto X^{1/2\{1-\sqrt{A}-e\}} \quad (16)$$

For determining the growth index of the perturbations we use the growing mode  $\delta_{m+}$  (equation (16)) and substitute into equation (12). It is well known the biasing effect in galaxy formation, i. e., the relative perturbations in the galaxy field and the matter field, on a point-by-point basis, are not equal:

$$\frac{\delta n}{n}(x) = b \frac{\delta \rho}{\rho}(x) \quad (17)$$

where  $n$  refers to the galaxy number density and  $b$  is the bias parameter. The parameter  $\beta = \frac{f}{b}$  relates the growth rate  $f$  of the perturbations (and hence the velocity field of the galaxy motions) with the density bias  $b$ . In this sense astrophysicists speak on a velocity/density comparison. Indeed, a compelling agreement is seen to exist between the velocity and density fields, which offers one possible test for the gravitational instability picture for the origin of structure [15].

The 2dFGRS has obtained redshift of a quarter million galaxies with an effective depth of  $z = 0.17$ . From a precise measurement of the clustering, a value of  $\beta = 0.43 \pm 0.07$  has been

determined [12]. The bias parameter has been measured by computing the bispectrum of the 2dFGRS, it is  $b = 1.04 \pm 0.11$  [14]. Combining the measurements of  $\beta$  and  $b$ , the growth index at  $z = 0.17$  can be estimated: it is  $f(z = 0.17) = 0.45 \pm 0.06$  [14]. The intermediate parameters in equation (16) for the growing mode of density perturbations can be recast into the parameter spaces of our models  $(\gamma, k, \varepsilon)$  and  $(\gamma, \lambda, \varepsilon)$ , respectively. After using equation (12), this results that the growth rate  $f$  will have the same parametric dependence. Now we use this fact to additionally constrain these spaces.

## IV. OBSERVATIONAL CHECK

### A. Model E

As said above, in the original paper  $(\gamma, k, \varepsilon)$  had been already constrained to  $(\gamma = 1, k > 0, 0 \leq \varepsilon \leq 0.045)$ . Applying equation (12) does not give further constrain on  $\varepsilon$ , but it does on  $k$ , as seen in Figure 1. Assuming a flat universe, the parameter  $\varepsilon = \Omega_\varphi(\infty)$ , is the amount

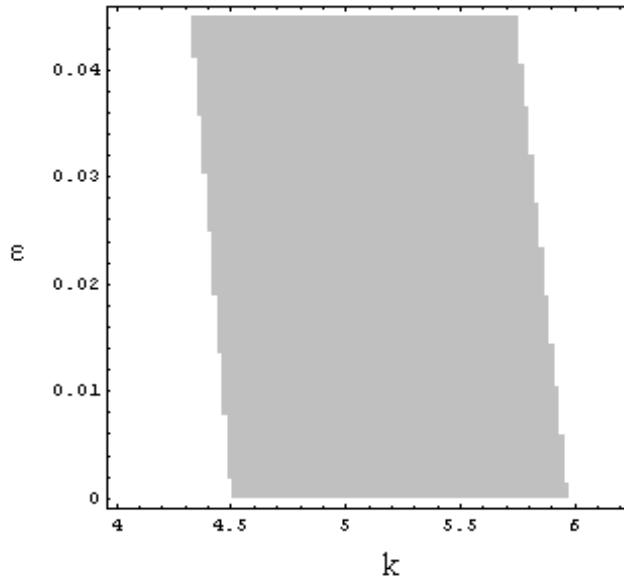


FIG. 1. Parameter Space for the model based on Einstein gravity. The parameter  $k$  is now constrained to a rather narrow region

of dark energy in the very early universe ( $z \sim \infty$ ). It is known that an appreciable amount of dark energy at that epoch would imply an expansion fast enough to prevent the formation

of structure at  $z \sim 3$ , but this parameter had been already constrained in the original paper, so it is not problematic that now it has not been additionally constrained. On the other hand, as showed in [3], the condition (1) acts like an interesting selection principle that, amongst the possible critical points, preserves only the attractor-like solutions. In this case, the velocity/density comparison allows to locate  $k$  in a rather narrow region, thus acting as a stringent selector of attractor-like solutions. Two things are to be said. Firstly, Figures 1 shows the joint region  $(k, \varepsilon)$  satisfying the velocity/density comparison, for  $k$  alone we might have a wider variation. Secondly, though narrow the interval, still we can speak of  $k$  as a selector of a class of solutions. Of course, with this models the authors don't pretend to definitely solve the fine tuning issue arising in cosmology.

## B. Model BD

In the figure 2 we show the parameter space that we obtained for the model based on Brans-Dicke gravity. In this figure the region inside straight line is the parameter space obtained in [3], while the one inside the dashed line is the parameter space obtained considering the perturbation growth. Then the shaded region gives the new parameter space, obeying all constrains considered so far. The parameters  $(\lambda, \varepsilon)$  of this model have a similar physical interpretation of  $(k, \epsilon)$ . In this case, both parameters were already considerably constrained from the original observations. The consideration of the growth rate of perturbations now gives additional restrictions, as seen from Figure 2. The above interpretation of  $k$  as a stringent selector of a class of attractor-like solutions also holds for  $\lambda$ .

## V. CONCLUSIONS

In the models presented here we found that the study of perturbation growth is a good tool to constraint the parameter space. Research on the origins and evolution of the large-scale of the universe is one of the hottest topics in cosmology. In this work, we have used the relation between the peculiar velocity field of the galaxies, the growth rate of perturbations

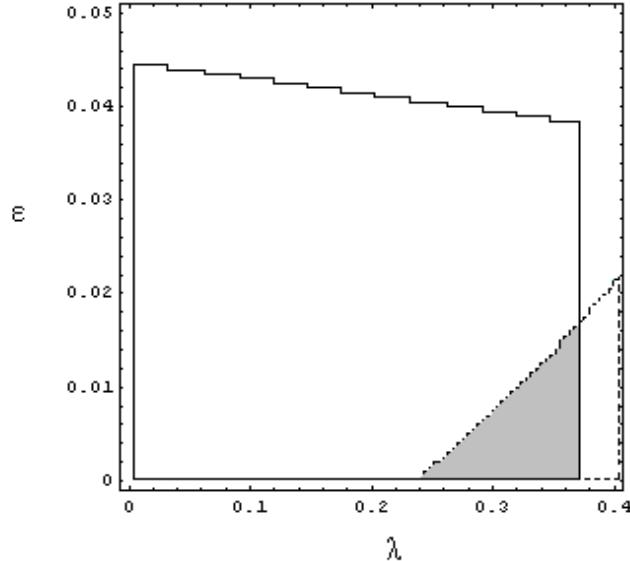


FIG. 2. Parameter Space for BD gravity. The region inside the straight line marks original constrains, while the region inside the dashed line signals the constrain from perturbations growth and the density bias in galaxy formation to make another step in the observational check of two quintessence models, resulting in a further constrain on the parameter space of the models. We plan in the oncoming future proceed this works using another cosmological probes, like CMB, for instance.

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